Difference equation

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Definition 1. A *difference equation* gives the relationship between an *independent variable* and a *dependent* variable, which changes at fixed, equally spaced intervals in time. The *order* of a difference equation is the number of time intervals spanned within the equation.

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Example 1. In the difference equation $y_{t+1} = 1.1y_t$, an independent variable is the time t while the dependent variable is the income y. The order is the span of t intervals within the equation, which in this case is $(t+1) - t = 1$.

Theorem 1. The general solution of a homogeneous first-order difference equation is of the form $y_t = Aa^t$, where t and y are respectively the independent and dependent variables.

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Note 1. Given a homogeneous first-order difference equation of the form $y_{t+1} - by_t = 0$ and some conditions, we may find the parameters a and A of our general solution $y_t = Aa^t$ by first finding a by substitution of this general solution for t and $t+1$ in the difference equation, and then find A from the conditions given.

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Theorem 2. The stability of the solution to a difference equation in general form $y_t = Aa^t$ is,

range of a	time path of y_t	*solution*	*time path*
$-\infty < a < -1$	$a^t \to \pm\infty$	unstable	alternates
$-1 < a < 0$	$a^t \to 0$	stable	alternates
$0 < a < 1$	$a^t \to 0$	stable	tends to zero
$1 < a < \infty$	$a^t \to \infty$	unstable	tends to infinity

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Theorem 3. The solution of a non-homogeneous difference equation is the sum of a complementary function and a particular integral, that is $y_t = y_c + y_p$. The *complementary function* is the solution of the homogeneous part of the difference equation. The *particular integral* is a function which satisfies the full difference equation.

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Note 2. The general form of the particular integral is deduced from the right-hand side of the difference equation. In particular, if c and b are constants,

right-hand side	*general form of particular integral*
c	$y_p = k$
cb^t	$y_p = kb^t$

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Bibliography

Teresa Bradley and Paul Patton. *Essential mathematics for economics and business.* 2^{nd} ed. 2002(1998)